

Hong Kong Mathematics Olympiad (2024/25)
Heats – Group Event
香港數學競賽 (2024/25)
初賽團體項目

INSTRUCTIONS

1. Before the start of the examination, you should follow the announcement to first write your School ID, School name, seat Number and your name, in the appropriate space provided on the answer sheet.
宣布開考前，學生須遵照司儀的指示在答題紙適當位置填寫你的學校編號、學校名稱、座位編號及你的姓名。
2. This paper consists of **TWO** sections, A and B.
本試卷分兩部分，即甲部和乙部。
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in the attached Answer Sheet. Do not write in the margins. Answers written in the margins will not be marked.
本試卷各題均須作答，答案須寫在隨附的答題紙中預留的空位內。不可在邊界以外位置書寫。寫於邊界以外的答案，將不予評閱。
4. Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
除特別指明外，所有答案須以數字的真確值表達並化至最簡。
5. No approximation is accepted.
不接受近似值。
6. Unless otherwise specified, all working **NEED NOT** be shown.
除特別指明外，不須列出所有算式。
7. The diagrams in this paper are not necessarily drawn to scale.
本試卷的附圖不一定依比例繪成。
8. No extra time will be given to candidates for filling in your School ID, School name, seat Number and your name after the ‘Time is up’ announcement.
司儀宣布停筆後，考生不會獲得額外時間填寫你的學校編號、學校名稱、座位編號及你的姓名。

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除特別指明外，所有答案須以數字的真確值表達，並化至最簡。

不接受近似值。

所有附圖不一定依比例繪成。

Part A

甲部

1. Let α , β and γ be the roots of $2x^3 - 3x^2 - 4x - 5 = 0$. Find the value of $(\alpha+1)(\beta+1)(\gamma+1)$.

設 α 、 β 及 γ 為 $2x^3 - 3x^2 - 4x - 5 = 0$ 的根。求 $(\alpha+1)(\beta+1)(\gamma+1)$ 的值。

2. Find the sum of all real solution(s) satisfying $(x^2 - 9x + 19)^{7x^2 + 6x - 1} - 1 = 0$.

求所有滿足於 $(x^2 - 9x + 19)^{7x^2 + 6x - 1} - 1 = 0$ 的實數解之和。

3. Let $f_1(x) = \frac{x}{1-x}$ and $f_n(x) = f_{n-1}(f_1(x))$, where $n = 2, 3, 4, \dots$. Find the value of $f_{2025}(2)$.

設 $f_1(x) = \frac{x}{1-x}$ ，及 $f_n(x) = f_{n-1}(f_1(x))$ ，其中 $n = 2, 3, 4, \dots$ 。求 $f_{2025}(2)$ 的值。

4. Given that

$$A = \sin^2(1^\circ) + \sin^2(3^\circ) + \sin^2(5^\circ) + \dots + \sin^2(20240277^\circ) + \sin^2(20240279^\circ),$$

$$B = \cos^2(1^\circ) + \cos^2(3^\circ) + \cos^2(5^\circ) + \dots + \cos^2(20240277^\circ) + \cos^2(20240279^\circ),$$

Find the value of $A^2 - B^2$.

已知

$$A = \sin^2(1^\circ) + \sin^2(3^\circ) + \sin^2(5^\circ) + \dots + \sin^2(20240277^\circ) + \sin^2(20240279^\circ),$$

$$B = \cos^2(1^\circ) + \cos^2(3^\circ) + \cos^2(5^\circ) + \dots + \cos^2(20240277^\circ) + \cos^2(20240279^\circ),$$

求 $A^2 - B^2$ 的值。

5. In Figure 1, the circle passes through two vertices of the square and touches one side of the square. If the radius of the circle is 10, find the area of the square.

圖一中，一個圓形通過一個正方形的兩個頂點，且與該正方形的其中一邊相切。設該圓的半徑 10，求該正方形的面積。

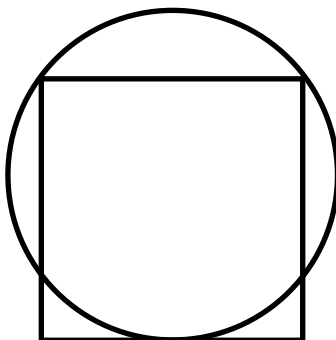


Figure 1

圖一

Part B

乙部

6. Given that $f(x) = \frac{9^x}{9^x + 3}$, find the value of $f\left(\frac{1}{2025}\right) + f\left(\frac{2}{2025}\right) + f\left(\frac{3}{2025}\right) + \dots + f\left(\frac{2024}{2025}\right)$.

已知 $f(x) = \frac{9^x}{9^x + 3}$ ，求 $f\left(\frac{1}{2025}\right) + f\left(\frac{2}{2025}\right) + f\left(\frac{3}{2025}\right) + \dots + f\left(\frac{2024}{2025}\right)$ 的值。

7. A positive integer X equals to the sum of the square of its four smallest positive factors, find the largest prime factor of X .

一個正整數 X 相等於它的四個最小正因數的平方和，求 X 的最大質因數。

8. In Figure 6, point P lies inside $\triangle ABC$ such that $\angle PAB = 12^\circ$, $\angle PBA = 18^\circ$, $\angle PCA = 30^\circ$ and $\angle PAC = 42^\circ$. Find $\angle PCB$.

圖 6 中，點 P 是 $\triangle ABC$ 內一點使得 $\angle PAB = 12^\circ$ 、 $\angle PBA = 18^\circ$ 、 $\angle PCA = 30^\circ$ 及 $\angle PAC = 42^\circ$ 。求 $\angle PCB$ 。

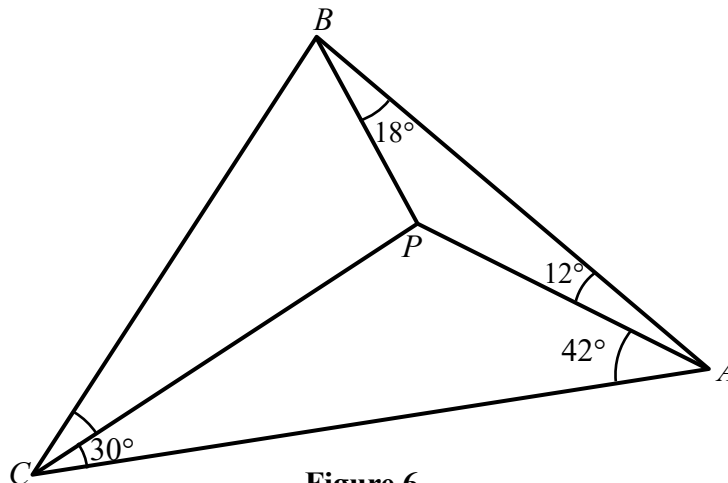


Figure 6

圖 6

9. Given that $0^\circ \leq \theta \leq 45^\circ$ and $16^{\tan^2 \theta} + 16 = 8(8^{\tan^2 \theta}) + 2(2^{\tan^2 \theta})$, find θ .

已知 $0^\circ \leq \theta \leq 45^\circ$ 及 $16^{\tan^2 \theta} + 16 = 8(8^{\tan^2 \theta}) + 2(2^{\tan^2 \theta})$ ，求 θ 。

10. In Figure 7, $ABCD$ is a square and $BCDE$ is a cyclic quadrilateral. If $AE \times CE = 28$ and $BE \times DE = 96$, find the perimeter of the square $ABCD$.

在圖七中， $ABCD$ 是一個正方形及 $BCDE$ 是一個圓內接四邊形。若 $AE \times CE = 28$ 及 $BE \times DE = 96$ ，求正方形 $ABCD$ 的周界。

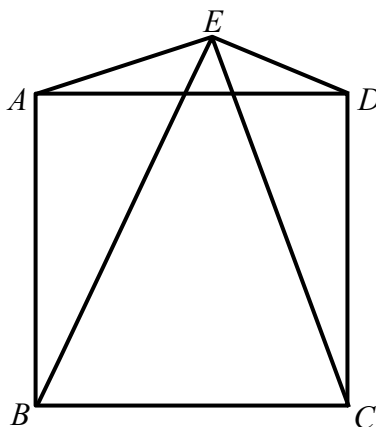


Figure 7

圖 七

END

完